## Semester One Examination, 2022

## Question/Answer booklet

# MATHEMATICS SPECIALIST UNIT 3

## Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet (retained from Section One)

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 50 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 90 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

| Markers use only | | |
|---|---|---|
| Question | Maximum | Mark |
| 8 | 8 | |
| 9 | 7 | |
| 10 | 7 | |
| 11 | 7 | |
| 12 | 8 | |
| 13 | 8 | |
| 14 | 10 | |
| 15 | 6 | |
| 16 | 8 | |
| 17 | 5 | |
| 18 | 8 | |
| 19 | 8 | |
| S2 Total | 90 | |
| S2 Wt (×0.7222) | 65% | |

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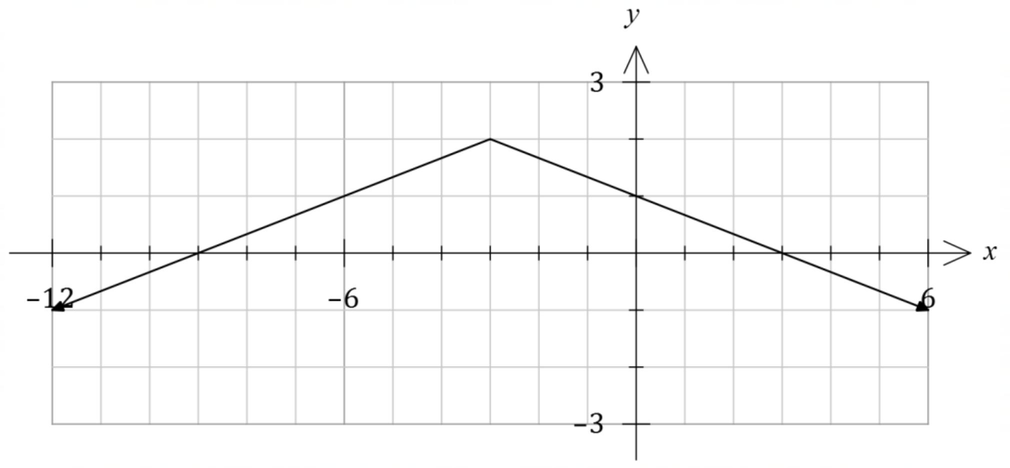
## Section Two: Calculator-assumed 65% (90 Marks)

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

**Question 8** **(8 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = a - |bx + c|$ and $a, b$ and $c$ are all positive constants.




(a) Determine the value of each of the constants $a, b$ and $c$. (3 marks)

(b) Using the graph, or otherwise, solve

(i) $f(x) = 1$. (1 mark)

(ii) $f(x) = |x| - 3$. (2 marks)

(iii) $3f(x) = |x - 3|$. (2 marks)

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**Question 9** **(7 marks)**

Point $C$ lies on a sphere with centre $O$, radius $r$ and diameter $AB$.

(a) Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. Use a vector method to prove that $AC$ is perpendicular to $BC$.
(4 marks)

(b) If the position vectors of $A, B$ and $C$ are $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} k \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$ respectively, determine the value of the constant $k$.
(3 marks)

**Question 10** **(7 marks)**

(a) Solve the equation $81z^4 + i = 0$, giving exact solutions in the form $r\,\mathrm{cis}\,\theta$ , $-\pi < \theta \leq \pi$. (4 marks)

(b) One solution of the equation $z^n = 1$, where $n$ is a positive integer, is $z = \mathrm{cis}\left(\frac{9\pi}{17}\right)$.
If $N$ solutions of the equation satisfy $0 < \arg(z) < \frac{\pi}{4}$, determine, with reasoning, the least value of $N$. (3 marks)

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**Question 11** **(7 marks)**

A small body is moving with constant velocity in space so that initially it is located at $(5, -7, -5)$ and four seconds later it is at $(13, -11, 7)$, where all dimensions are in metres.

(a) Determine a vector equation for the position of the small body at time $t$ seconds. (2 marks)

A laser beam shines along the line with equation $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z-16}{2}$

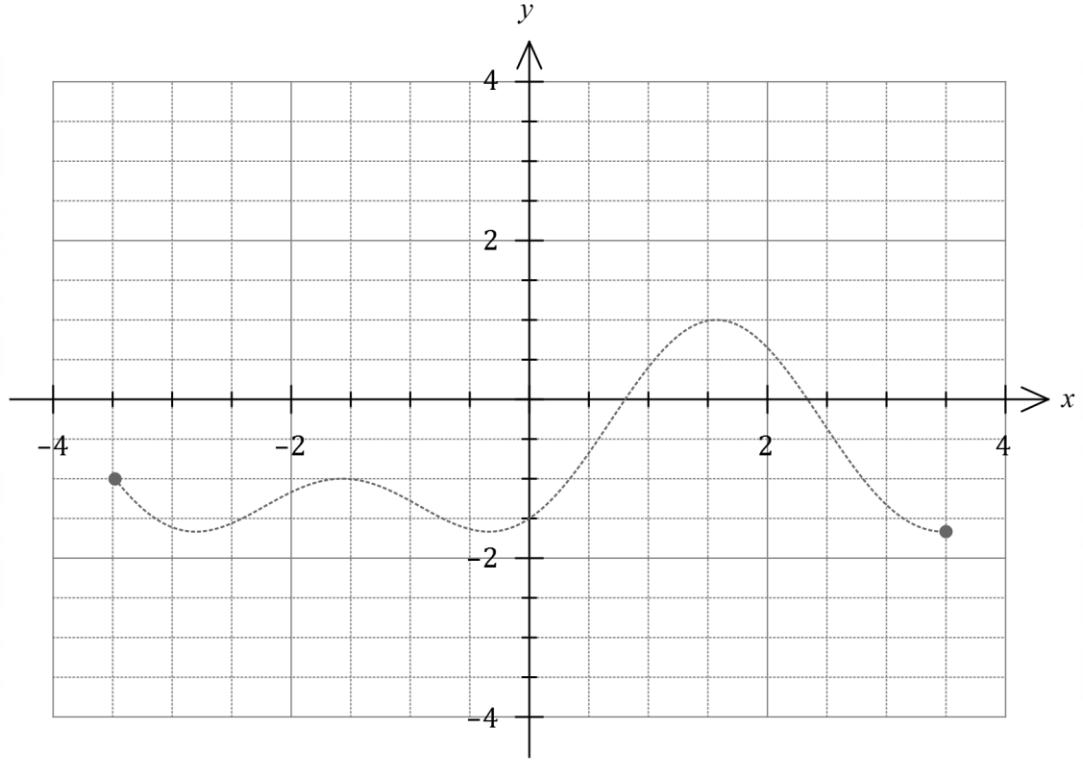
(b) Write the vector equation of this line in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. (1 mark)

(c) Show that the small body passes through the laser beam and state where this occurs. (4 marks)

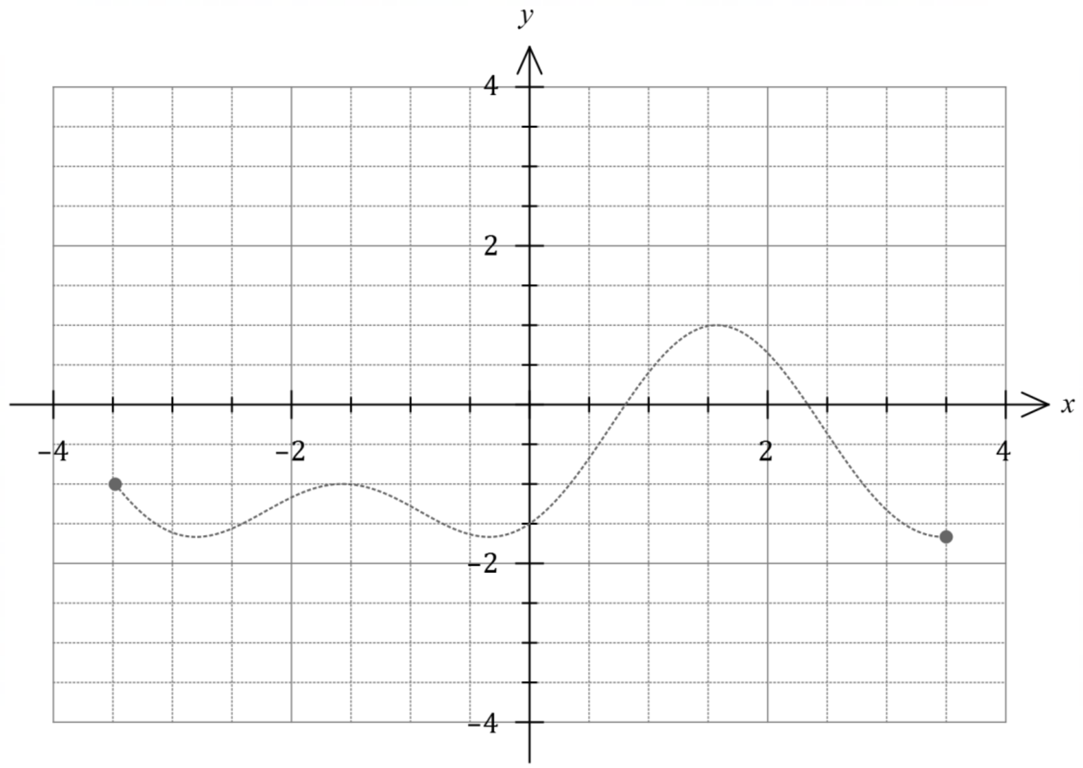
**Question 12** **(8 marks)**

In each part of this question, the dotted curve shown is the graph of $y = f(x)$.

(a) Sketch the graph of $y = |f(x)|$. (2 marks)




(b) Sketch the graph of $y = \frac{1}{f(x)}$. (4 marks)

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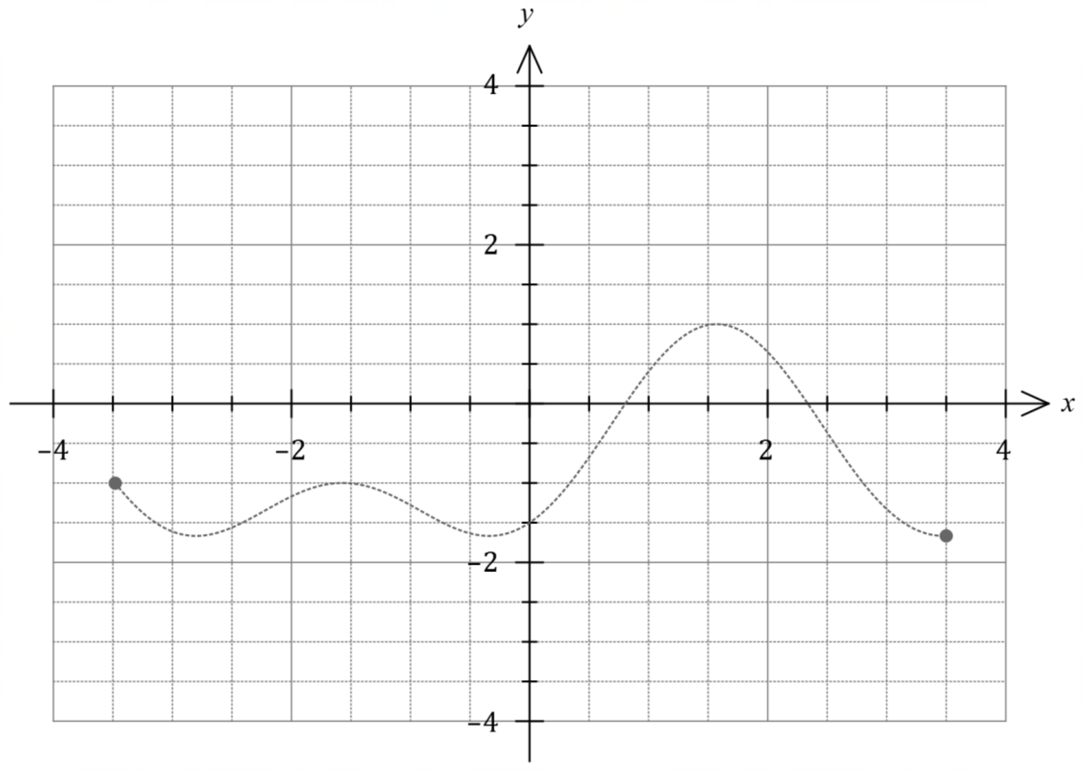
(c) Sketch the graph of $y = f(-|x|)$. (2 marks)

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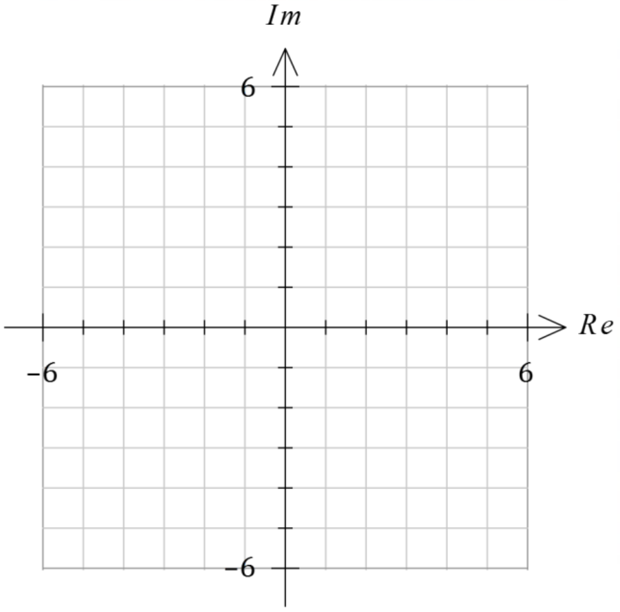
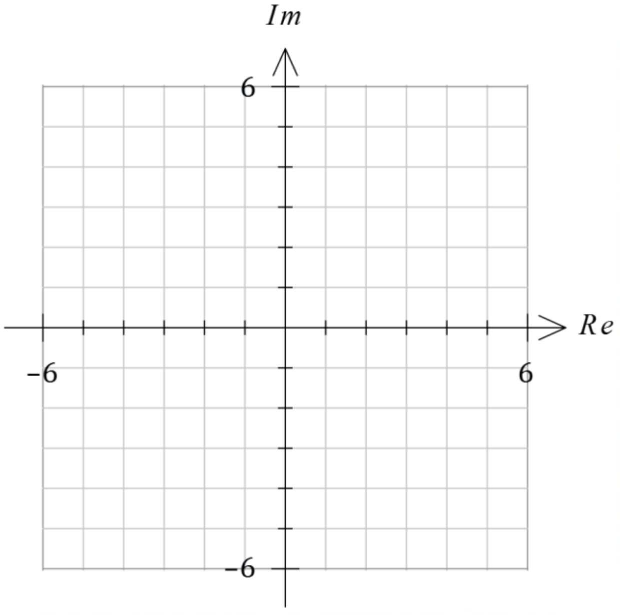
**Question 13** **(8 marks)**

(a) On the Argand planes below sketch the locus of the complex number $z = x + iy$ given by

(i) $|z + 3 - 4i| = |z - 2 + i|$. (3 marks)



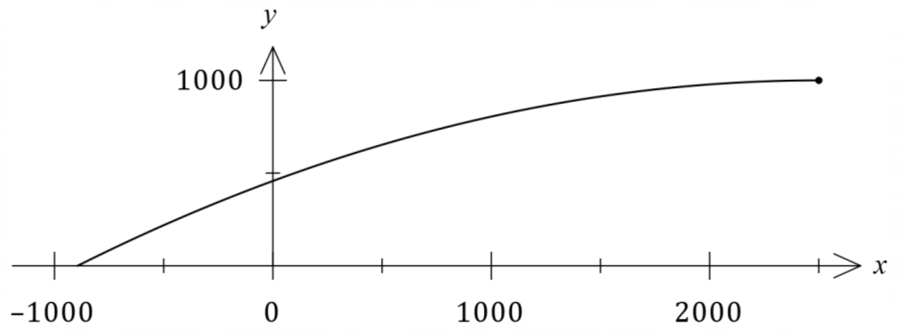

(ii) $|\bar{z} + 3i| \leq 3$. (3 marks)




(b) For the locus $|z + 3 - 4i| = |z - 2 + i|$ in part (a), determine the minimum value for $|z + 4i|$. (2 marks)

**Question 14** **(10 marks)**

An aeroplane flying at a constant altitude releases a bomb at $2500\mathbf{i} + 1000\mathbf{j}$ with an initial velocity of $-240\mathbf{i}$. The path of the bomb is shown below.




Assume there is no wind in the region, air resistance can be ignored and the only acceleration acting on the bomb is $-10\mathbf{j}$ ms$^{-2}$ due to gravity.

(a) Use the acceleration vector of the bomb to clearly deduce that its position vector at time $t$ seconds after release is $\mathbf{r}(t) = (2500 - 240t)\mathbf{i} + (1000 - 5t^2)\mathbf{j}$. (3 marks)

(b) Determine the speed of the bomb $6$ seconds after it is released. (2 marks)

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Three seconds after the bomb is released, a projectile is launched from the origin with a speed of $v_0$ at an angle of elevation of $\theta°$ to intercept it at a height of $680$ m.

The position vector of the projectile $T$ seconds after its launch is

$$\mathbf{r}(T) = (v_0 \cos(\theta)\, T)\mathbf{i} + (v_0 \sin(\theta)\, T - 5T^2)\mathbf{j}.$$

(c) Determine the value of $v_0$ and the value of $\theta$ so that the projectile intercepts the bomb. (5 marks)

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**Question 15** **(6 marks)**

Points $P$, $Q$ and $R$ lie in plane $\Pi$ with position vectors $\begin{pmatrix}2\\1\\5\end{pmatrix}$, $\begin{pmatrix}-2\\-2\\5\end{pmatrix}$ and $\begin{pmatrix}0\\0\\4\end{pmatrix}$ respectively.

(a) Determine the vector equation for plane $\Pi$ in the form $\mathbf{r} \cdot \mathbf{n} = k$. (3 marks)

The equation of line $L$ is $\mathbf{r} = \begin{pmatrix}3\\5\\-2\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}$.

(b) Determine, if possible, where line $L$ intersects with plane $\Pi$. If not possible, explain why not. (3 marks)

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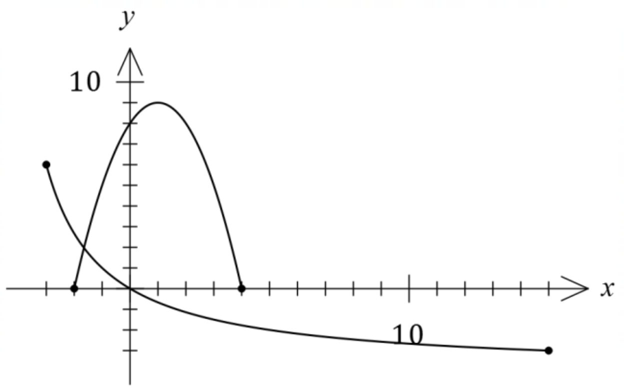
**Question 16** **(8 marks)**

The graphs of $y = f(x)$ and $y = g(x)$ are shown at right.




The functions are defined by

$$f(x) = \frac{-4x}{x+5}, \qquad -3 \le x \le 15$$

and

$$g(x) = -x^2 + 2x + 8, \qquad -2 \le x \le 4.$$

(a) Explain why the inverse of $g$ is not a function. (1 mark)

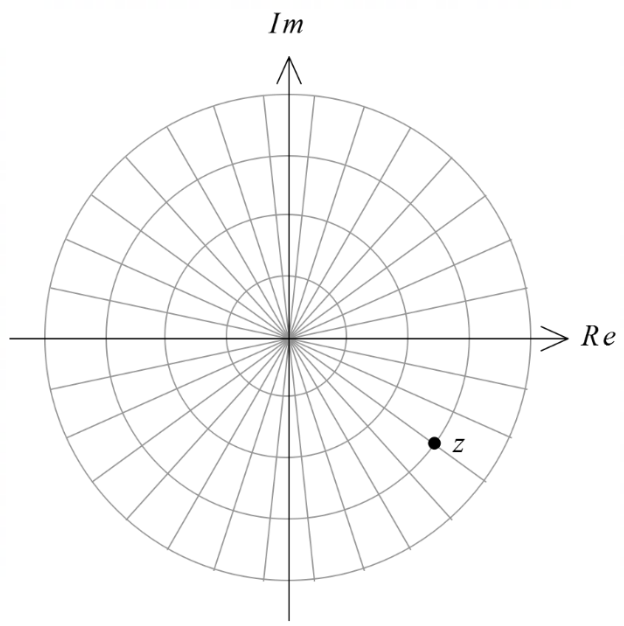
(b) Determine the definition for the inverse of $f$. (3 marks)

(c) Determine $g \circ f(-1)$. (1 mark)

(d) Determine the domain for the function $g \circ f(x)$. (3 marks)

**Question 17** **(5 marks)**

The complex number $z$ is shown on the Argand diagram below and $w = \cos\left(-\frac{4\pi}{15}\right) + i\sin\left(-\frac{4\pi}{15}\right)$.




(a) Describe the geometric transformation performed by $w$ when another complex number is multiplied by it, and plot and label $zw$ on the Argand diagram. (2 marks)

(b) Plot and label the complex number $zw^{2022}$ on the Argand diagram. (3 marks)

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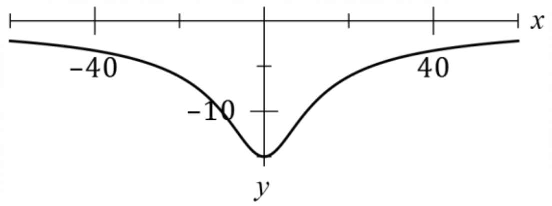
**Question 18** **(8 marks)**

The path of a small submersible moving below the surface of the sea (the $x$-axis) is shown in the diagram, where $t$ is the time in seconds and $0 < t < 3\pi$.
The position vector of the submersible is

$$\mathbf{r}(t) = 9\cot\left(\frac{t}{3}\right)\mathbf{i} - 15\sin\left(\frac{t}{3}\right)\mathbf{j}\ \text{m}.$$




(a) State, with reasoning, whether the submersible is moving from left to right or from right to left. (2 marks)

(b) Determine the Cartesian equation for the path of the submersible. (3 marks)

(c) Determine the distance travelled by the submersible when its depth below the surface is at least 7.5 metres, correct to the nearest centimetre. (3 marks)

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**Question 19** **(8 marks)**

The vector equation of sphere $S_1$ is $|\mathbf{r} - (-17\mathbf{i} + 15\mathbf{k})| = 22$. The position vector of the centre of sphere $S_2$ is $10\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ and the position vector of a point that lies on $S_2$ is $8\mathbf{i} + 12\mathbf{j} + 6\mathbf{k}$.

(a) Determine the Cartesian equation of sphere $S_2$. (2 marks)

(b) The equation of line $L_1$ is $\mathbf{r} = -12\mathbf{i} + 24\mathbf{j} + 9\mathbf{k} + \lambda(3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$. Show that $L_1$ is tangential to $S_1$ and determine the position vector for the point of tangency. (4 marks)

(c) Show that $S_1$ and $S_2$ are tangential. (2 marks)



**End of questions**

Supplementary page

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